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De Sitter holography and the cosmic microwave background

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Abstract: We interpret cosmological evolution holographically as a renormalisation group flow in a dual euclidean field theory, as suggested by the conjectured dS/CFT correspondence. Inflation is described by perturbing around the infra-red fixed point of the dual field theory. The spectrum of the cosmic microwave background radiation is determined in terms of scaling violations in the field theory. The dark energy allows similar, albeit less predictive, considerations. We discuss the cosmological fine-tuning problems from the holographic perspective.

Keywords: AdS-CFT and dS-CFT Correspondence, Cosmology of Theories beyond the SM.

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1. Introduction

Astronomical observations indicate that the universe is currently accelerating. The simplest interpretation of this data is that we are entering an epoch dominated by a positive cosmological constant, a de Sitter phase, which will persist eternally. The Cosmic Microwave Background anisotropy and other measurements pertaining to the very early universe similarly suggest an epoch of rapid acceleration in our distant past. The history of the universe may thus be interpreted as the interpolation between two de Sitter phases. The traditional framework to interpret this evolution is to hypothesize some medium, the inflaton field or quintessence, and design the dynamics of the corresponding scalar field so it dominates at early and late times, respectively. Recently, a more radical interpretation was proposed: the cosmic evolution is *holographic*; i.e. it can be usefully analyzed in terms of a dual field theory [1, 2]. This paper develops the holographic proposal further.

A holographic duality has been established only for anti-de Sitter space (AdS) [3], but it is plausible that many other backgrounds similarly allow holographic representations, particularly in view of the area law for gravitational entropy. In the case of AdS the dual theory is a conformally invariant gauge theory, situated at the AdS boundary at large radius. The symmetries of de Sitter space (dS) suggest that their duals are conformal field theories (CFT's) as well; these theories are euclidean and located at asymptotically early (or late) times [4, 5]. If the dS/CFT correspondence is correct, the cosmological evolution interpolating between two de Sitter-phases can be naturally interpreted as the renormalization group flow linking two CFT's (analogous flows were considered in AdS [6]–[9]). Then there is one underlying quantum field theory, with the future de Sitter phase governed by the ultraviolet (UV) fixed point, and the past corresponding to the infrared (IR) fixed point. The increasing entropy of the expanding universe is interpreted holographically as the "integrating in" of the reverse renormalization group flow.

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The purpose of this paper is to make this abstract scenario more concrete. Our main example is the anisotropy of the cosmic microwave background (CMB) radiation. We find that holography gives the same predictions as standard slow-roll inflation; in fact, there is a simple dictionary between the holographic model and the standard approach using a single spacetime scalar field. However, the holographic interpretation is different: the basic input is the dual quantum field theory, rather than the inflationary potential; so now scaling violations are interpreted in terms of the β-function and the anomalous dimension. One reason this is interesting is that it gives a new perspective on the fine-tuning problems introduced by inflation, and on the cosmological constant problem. A different approach to interpreting the CMB in terms of critical scaling behavior can be found in [10].

Another motivation for studying the holographic formulation is that it may provide a phenomenologically viable path to microphysics.[1] It is often presumed that the inflationary potential will be the interlocutor, inferred in some detail from observations as well as derived from a low energy limit of string/M-theory. It seems to us no less fanciful to envision interpreting observations in terms of scaling violations and compare them with a microscopic understanding of the dual field theory.

In the simplest viable models of inflation, there is a single inflaton field which dominates the energy density [16, 17]. The quantum fluctuations of this field are ultimately responsible for the CMB anisotropy. The detailed spectrum depends on small departures from pure de Sitter space and is conventionally parametrized in terms of the slow-roll parameters ϵ and η. In the dual interpretation the inflationary epoch is reinterpreted as a quantum field theory close to its IR fixed point. The CMB spectrum now depends on the scaling violations of the field theory and is simply parametrized by the β-function and the anomalous dimension λ. Perturbative quantum field theory gives $\beta \ll \lambda$ corresponding to the prediction $\epsilon \ll \eta$.

In section 2, we first review standard inflationary cosmology and then summarize the conjectured dS/CFT correspondence. Next, we explain how the bulk physics determines properties of the CMB, and show how this is reinterpreted in the dual theory. Of course, we do not as yet understand all aspects of the dS/CFT duality, but for our purposes a few simple features suffice. The most conservative view is that we simply reorganize the standard inflationary picture in a way inspired by dS/CFT, but ultimately interpretable in terms of symmetries. This point of view could be useful phenomenologically even if no holographic dual exists. In the remainder of the paper we will simply assume the dS/CFT correspondence and take the view that we are deriving some of its consequences.

In section 3 we introduce a concrete quantum field theory realization of the central features assumed in our general discussion. We explicitly compute the CMB spectrum in this toy model of the early universe. We conclude the paper in section 4 with a discussion of some standard problems in inflationary cosmology, such as the cosmological constant problem, the fine-tuning of the inflaton potential, and the initial value problem. We translate these problems to the dual theory and explain some potential advantages of the holographic perspective.

[1] Other recent attempts to connect cosmology and microphysics appear in [11]–[15].

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2. Holographic cosmology

2.1 FRW cosmology

We consider 4d Einstein gravity coupled to scalars ϕ^I *via*

$$\mathcal{L} = \frac{1}{2\kappa^2}R - \frac{1}{2}G_{IJ}\partial_\mu\phi^I\,\partial^\mu\phi^J - V\left(\phi^I\right), \tag{2.1}$$

where $\kappa^2 = 8\pi G = 1/{M_p}^2$ and G_{IJ} is the metric on the moduli space of the scalars ϕ^I. The potential $V(\phi^I)$ is assumed to have de Sitter extrema. Using the standard spatially flat FRW metric

$$ds^2 = -dt^2 + a(t)^2\left(dr^2 + r^2 d\Omega_2^2\right), \tag{2.2}$$

and assuming spatial isotropy $\vec{\nabla}\phi^I = 0$, the equations of motion are given by

$$\ddot{\phi}^I + 3H\dot{\phi}^I + G^{IJ}\frac{dV}{d\phi^J} = 0\,, \tag{2.3}$$

$$H^2 = \frac{1}{3}\kappa^2\rho\,, \tag{2.4}$$

$$\dot{H} = -\frac{1}{2}\kappa^2\left(\rho + p\right), \tag{2.5}$$

where the Hubble parameter H is $H \equiv \dot{a}/a$, the dot representing a derivative with respect to the coordinate time t, and the density ρ and pressure p are given by

$$\begin{aligned} \rho &= \frac{1}{2}G_{IJ}\dot{\phi}^I\dot{\phi}^J + V\left(\phi^I\right), \\ p &= \frac{1}{2}G_{IJ}\dot{\phi}^I\dot{\phi}^J - V\left(\phi^I\right). \end{aligned} \tag{2.6}$$

The equations of motion give

$$\frac{3}{4}\kappa^2(\omega + 1) = H^{-2}G^{IJ}\frac{dH}{d\phi^I}\frac{dH}{d\phi^J}\,, \tag{2.7}$$

for the equation of state parameter $\omega \equiv p/\rho$. Also recall that one of the equations of motion is redundant, e.g. (2.3) and (2.4) imply (2.5).

In the following we drop the index I and consider a single scalar with canonical kinetic energy; the generalization back to multiple scalars is straightforward.

2.2 The dS/CFT correspondence

We now want to reinterpret this 4d bulk cosmology in terms of a dual 3d euclidean quantum field theory. The idea is that the scalar field breaks the de Sitter symmetry in the bulk and this will correspond to broken scale invariance in the dual theory.

According to the dS/CFT correspondence a scalar field ϕ with asymptotic value $\phi_0(\vec{x})$ is dual to an operator $\mathcal{O}$. This means the dual theory is perturbed away from its conformal fixed point

$$\mathcal{L} = \mathcal{L}_{\mathrm{CFT}} + g\mathcal{O}\,, \tag{2.8}$$

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where the coupling $g = \kappa\,\phi_0$. The QFT scale parameter is $\mu \propto a$, mapping bulk UV (early times) to field theory IR and *vice versa*. With this map the QFT β-function translates into the bulk as

$$\beta \equiv \frac{\partial g}{\partial \log \mu} = \frac{\partial}{\partial \log a}\kappa\phi = -\frac{2}{\kappa H}\frac{dH}{d\phi}\,. \tag{2.9}$$

The last step employed the FRW equations (2.4), (2.5).

To proceed we assume that the potential has a mass term $\partial_\phi^2 V|_{\phi=0} = m^2 \neq 0$. Then the Klein-Gordon scalar equation (2.3) with $a(t) \sim e^{H_0 t}$ as either $t \to \infty$ or $t \to -\infty$ determines the asymptotic wave function $\phi = \phi_0(\vec{x})e^{\lambda H_0 t}$ where

$$\lambda^2 + 3\lambda + \frac{m^2}{H_0^2} = 0\,, \tag{2.10}$$

giving two solutions for λ

$$\lambda_\pm = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - \frac{m^2}{H_0^2}}\,. \tag{2.11}$$

A similar relation (with $m^2 \to -m^2$) is standard in the AdS/CFT correspondence [18]. To interpret the parameter λ note that, in the asymptotic de Sitter space, the wave function transforms as $\phi \to \phi e^{\lambda H_0 \Delta t}$ under a time translation $t \to t + \Delta t$. But, according to the metric (2.2), this can also be viewed as a rescaling of the spatial slice $\vec{x} \to (\mu/\mu_0)\vec{x}$ where $\mu/\mu_0 = e^{\Delta t H_0}$. Thus $\phi_0 \to (\mu/\mu_0)^\lambda \phi_0$ under rescaling within the spatial slice; so λ is the anomalous scaling dimension of the dual operator $\mathcal{O}$. We can relate it to the beta function as

$$\lambda = \frac{\partial \log \phi_0}{\partial \log a} = \frac{\partial \log g}{\partial \log \mu} = \frac{\beta}{g}\,. \tag{2.12}$$

For each field there are two values of λ, given by (2.11). The interpretation of λ in terms of the asymptotic behavior of the wave function $\phi_\pm \sim e^{\lambda_\pm H_0 t}$ shows that λ_+ corresponds to infinite asymptotic energy $E = \int \dot{\phi}^2 a^3 d^3x = \infty$; in contrast, λ_- is a finite energy perturbation $E < \infty$. It is therefore natural to interpret the λ_+ as a deformation of the dual field theory whereas λ_- corresponds to an excitation of the theory without changing the underlying lagrangian. This interpretation is standard in the AdS/CFT correspondence [19]. Since the energetics is similar in de Sitter space we expect the result to hold here as well.[2] In our application we want to consider the RG-flow between different theories. This will require deforming the theory and thus considering λ_+.

Let us now focus on the IR theory at $t \to -\infty$, the traditional inflationary regime. The approach to an IR fixed point is described by an *irrelevant* deformation of the IR theory. The field theory expectation is thus for a positive anomalous dimension $\lambda_+^{IR} > 0$. According to (2.11) this corresponds to $m^2 < 0$, a tachyon mass. This is expected from the bulk point of view as well: the scalar field rolls down a hill from a maximum.

[2]The distinction usually made is actually between *normalizable* and *non-normalizable* states. In bulk the standard norm is the Klein-Gordon norm, but in de Sitter space this norm vanishes because the wave-functions $\phi_\pm \sim e^{\lambda_\pm H_0 t}$ are real. Using the asymptotic energy as criterion attempts to circumvent this difficulty.

The approach to the IR fixed point of a quantum field theory is generally described by many irrelevant operators. However, the final approach is dominated by the operator(s) of smallest dimension because this will have survived the RG flow the longest. If this operator is nearly marginal, with $\lambda_+^{IR} \ll 1$, then the slow-roll conditions for inflation are satisfied in the bulk, as we will show in the next subsection.

The assumption of a nonvanishing mass of the bulk scalar is significant for the preceding discussion. The corresponding field theory statement is $\beta \propto g$ for small coupling, as follows from (2.12) with constant λ. This behavior corresponds to the β-function of a coupling with classical mass dimension, a common behavior in three dimensions. If we want to consider instead couplings with $\beta \propto g^n$ for $n > 1$ the corresponding bulk field has $m^2 = 0$ and the anomalous dimension λ_+ of the dual operator vanishes at the fixed point. In this case (2.9) can be inverted to determine the spacetime potential as $V \propto \phi^{n+1}$ for small ϕ. In perturbative QFT λ always satisfies (2.12); so we *define* λ as the g dependent function

$$\lambda \equiv \frac{\beta}{g}, \tag{2.13}$$

away from the fixed point. Then our considerations will be valid for these more general theories as well.

To complete our dictionary we introduce the holographic c-function. This function decreases along the RG-flow towards the IR and thus provides a precise formulation of the heuristic idea that the RG-flow integrates out degrees of freedom. The c-function has not yet been firmly established in QFT above two dimensions but the evidence in its favor is substantial [20]. Importantly, there is a natural candidate for a holographic interpretation in AdS [7] as well as in de Sitter [1]; it is given in 4 dimensions by

$$c \equiv \frac{1}{\kappa^2 H^2}. \tag{2.14}$$

The FRW equation (2.5) gives $\dot{H} < 0$ for matter satisfying the null energy condition (e.g. $p + \rho > 0$) so $c(t)$ increases in time. In cosmology it expresses the increasing entropy of the expanding universe; in QFT it represents the "integrating in" along the RG-flow towards the UV fixed point. De Sitter RG-flows and the holographic "c-theorem" were recently discussed in [21].

2.3 The slow-roll parameters

Spacetime inflates when the cosmological evolution is dominated by the potential energy of the scalar (for reviews on inflation see [22]–[25]) . This situation arises when $\dot{\phi}^2 \ll V(\phi)$ and $|\ddot{\phi}| \ll |3H\dot{\phi}|, |\partial_\phi V|$. It is convenient to introduce the slow roll parameters[3] [22]

$$\epsilon \equiv -\frac{\partial \log H}{\partial \log a} = \frac{2}{\kappa^2}\left(\frac{1}{H}\frac{dH}{d\phi}\right)^2, \tag{2.15}$$

$$\eta \equiv -\frac{\partial \log \frac{\partial H}{\partial \phi}}{\partial \log a} = \frac{2}{\kappa^2}\frac{1}{H}\frac{d^2 H}{d\phi^2}, \tag{2.16}$$

[3] Our slow-roll parameters are denoted ϵ_H and η_H in [22]. Another set of commonly used parameters are related to ours as $\epsilon_{\text{here}} = \epsilon_{\text{there}}$ and $\eta_{\text{here}} = \eta_{\text{there}} - \epsilon_{\text{there}}$ in the slow-roll limit.

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and write these conditions as

$$|\epsilon|, |\eta| \ll 1\,. \tag{2.17}$$

Inflation ends precisely when the slow-roll conditions are violated. From then on the cosmological evolution will be dominated by the kinetic energy of the scalar and by matter and radiation.

We now want to express the slow-roll parameters in terms of the β-function (2.9) and the anomalous dimension λ (2.13), the natural parameters in the dual field theory. Comparing (2.9) and (2.15) gives

$$\epsilon = \frac{1}{2}\beta^2\,. \tag{2.18}$$

The other slow-roll parameter η (2.16) can be written as

$$\eta = \frac{1}{2}\beta^2 - \frac{1}{\kappa}\frac{d\beta}{d\phi}\,. \tag{2.19}$$

In the vicinity of the fixed point at $g = 0$ (2.12) then gives

$$\eta = -\lambda\,. \tag{2.20}$$

When λ vanishes close to a fixed point with $\beta \propto g^n$ for $n > 1$ the relation becomes $\eta = -n\lambda$ to the leading order. The relations (2.18) and (2.20) provide the basic dictionary between scaling violations in QFT and inflation.

Inflation ends when the slow-roll parameters approach unity. The dual field theory interpretation is that the theory enters a complicated interacting regime with no useful perturbative description. A plausible scenario is that when $\lambda \ll 1$ is violated many hitherto negligible operators become important and so the simple scaling regime breaks down. The end of inflation can be modeled in detail but one must make specific assumptions about the complete field content and the corresponding β's and λ's, even away from the weakly coupled regime. We expect that such models would parallel the standard inflationary models.

2.4 Holography and the CMB

The main application of these ideas is to cosmological perturbations. A massless scalar field in de Sitter space has fluctuations at each wavenumber k, with magnitude $\delta\phi_k = H/2\pi$. This leads to density perturbations on every scale if the potential is sufficiently flat. The power spectrum of the scalar component of the spatial metric is

$$\mathcal{P}_{\text{scalar}} = \left(\frac{H}{\dot\phi}\right)^2 \left(\frac{H}{2\pi}\right)^2\Bigg|_{k=aH} \propto k^{n_S - 1}\,, \tag{2.21}$$

where the subscript $k = aH$ indicates that the expression is to be evaluated at the moment when the physical scale of the perturbation is equal to the Hubble radius. Perfect scale invariance corresponds to $n_S = 1$; and the scaling violations, due to the slow rolling of H and $\dot\phi$, become

$$n_S = 1 - 4\epsilon + 2\eta\,. \tag{2.22}$$

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The tensor component of the spatial metric also develops fluctuations, interpreted as graviton waves. Their power spectrum is

$$\mathcal{P}_{\rm grav} = 2\kappa^2 \left(\frac{H}{2\pi}\right)^2\bigg|_{k=aH} \propto k^{n_T} \,. \tag{2.23}$$

Perfect scale invariance now corresponds to $n_T = 0$; the scaling violations are

$$n_T = -2\epsilon \,. \tag{2.24}$$

A third observable is the ratio r of the tensor and scalar power[4]

$$r \equiv \frac{\mathcal{P}_{\rm grav}}{\mathcal{P}_{\rm scalar}} = 2\kappa^2 \left(\frac{\dot\phi}{H}\right)^2 = 4\epsilon \,. \tag{2.25}$$

The observables (2.21), (2.23), (2.25) are all expressed in terms of the slow-roll parameters ϵ and η; they can therefore also be expressed in terms of the dual β, λ, using (2.18) and (2.20). The results to the leading significant order are

$$n_S = 1 - 2\lambda \,, \tag{2.26}$$

$$n_T = -\beta^2 \,, \tag{2.27}$$

$$r = 2\beta^2 \,, \tag{2.28}$$

with $\beta \ll \lambda \ll 1$. The power of the tensor component is thus negligible; and the tilt of the scalar component is determined in terms of $\eta = -\lambda$ alone and because $\lambda > 0$ this implies a slightly red spectrum. In the standard notation $\epsilon \ll \eta \ll 1$ and $\eta < 0$, results also predicted by a large class of inflationary models [22, 24, 25].

We would like to understand these results from the QFT. The first step is to justify the use of perturbation theory. In QFT inflation ends in a strongly coupled regime; we want to compute correlators at this time with arguments corresponding to astronomically large distances. A huge RG transformation of $\mathcal{O}(10^{50-60})$ transforms this into correlators at more natural distances, and the RG flow also shifts the strong coupling towards the IR by a large amount, presumably bringing the coupling to the perturbative regime.[5] (However, we are not requiring the fixed point theory itself to be weakly coupled.)

The next step is to derive the fluctuation spectrum directly from QFT. This should also be possible if we assume certain details of the dS/CFT duality to hold. This is apparent, since the bulk fluctuations are determined by properties of correlation functions. The asymptotics of a bulk correlation function $\langle\phi\phi\rangle$ will be related to a boundary correlation function $\langle\mathcal{O}\mathcal{O}\rangle$, if ϕ is dual to $\mathcal{O}$. The momentum dependence of the boundary correlation function, close to the fixed point, is determined by the scaling dimension of $\mathcal{O}$ at the fixed point. Similar computations for the energy-momentum tensor in the CFT correspond to tensor fluctuations in bulk. We have not carried out this procedure explicitly; it would clearly be interesting to do so.

[4] The ratio of multipole moments $C_l^{(T)}/C_l^{(S)}$ is often quoted instead. It differs from this by an l-dependent numerical factor of geometric origin.

[5] This might not happen, despite the large scale transformation. Inflationary models with predictions different from ours might arise this way.

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2.5 Dark energy

Until this point our focus has been on the approach to the IR fixed point, i.e. the inflationary regime. In holographic cosmology the entire cosmological evolution can be interpreted in the dual field theory, at least in principle. The holographic variables are not useful throughout the RG-flow but they might simplify also close to the UV fixed point, i.e. in the regime dominated by dark energy.

The UV theory is perturbed by a relevant operator, inducing the RG flow. The simplest is to assume that one such operator dominates. This means there is some scalar field ϕ satisfying

$$\phi \sim e^{\lambda_+^{UV} H_0 t}\,; \qquad t \to \infty\,, \tag{2.29}$$

with[6] $-\frac{3}{2} < \lambda_+^{UV} < 0$. Equivalently, the mass of the UV scalar field is expected in the range $0 < \frac{m^2}{H_0^2} < (\frac{3}{2})^2$.

This type of model would be very similar to the quintessence model of dark energy [26, 27]. As for inflation one can translate the natural holographic variables, β and λ, to those customary in the quintessence literature. There one usually concentrates on the equation of state parameter ω, as well as the scalar mass m. λ is related to m^2 through (2.11) and ω is given by $3(\omega+1) = \beta^2$. We can now take over standard considerations from quintessence.

An important difference between inflation and dark energy is that the holographic perspective gives no clear motivation for considering a theory of a single scalar field close to the UV limit. Indeed, although we have used the same notation ϕ for the scalar field at both $t \to -\infty$ and $t \to \infty$ they are generally completely unrelated; the details of the UV and IR theories are very different. The only relation between these two regimes is that, as a matter of principle, they can be described by the same underlying quantum field theory.

3. The dual field theory

To this point we have not discussed in detail the precise nature of the duality between de Sitter space and conformal field theory [5, 28, 29]. Of course it could be a full-fledged duality, similar in nature to AdS/CFT. This view has conceptual [30, 31, 32] and practical difficulties: for example, de Sitter space does not seem to have simple realizations in string theory [33]–[39]. It has been established that the dS/CFT correspondence, if true, is not simply a continuation of the AdS/CFT correspondence [40, 41].

On the other hand, it is sometimes claimed that most or all gauge theories have gravitational duals [42]; this could be taken to mean that any euclidean CFT will have a dual de Sitter description. The matter content and other properties of the spacetime theory will then depend on the choice of CFT, but the background geometry should be de Sitter for all such CFT's. This optimistic view motivates us to consider a toy model, the simplest example of a euclidean theory in 3 dimensions with properties of the kind we have

[6] We only consider real λ_+^{UV}.

assumed in this paper. That the central charge $c_{IR} \sim \kappa^{-2}\Lambda_{IR}^{-1}$ is rather low for typical Λ_{IR} indeed gives evidence that the IR CFT can be quite simple. Even if this toy model is in fact not dual to de Sitter space it is an instructive illustration of our ideas.

Let us then consider the model

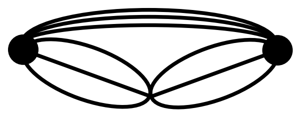
$$S_E = \int d^3x \, \left[\frac{1}{2}(\partial_\mu\varphi)^2 + \frac{g}{6!}\varphi^6\right]. \tag{3.1}$$

This has the virtue of being renormalizable. The φ^6 term is classically marginal but at the quantum level the coupling g is in fact marginally irrelevant in the IR. This is just what we want: the IR CFT is the trivial fixed point and scaling violations away from the fixed point are governed by perturbation theory.

The renormalization properties of this theory are quite simple in perturbation theory. Of course, for the action (3.1) to be valid near the fixed point, it must be finely tuned, as there are several relevant perturbations. (This feature could presumably be fixed in a simple supersymmetric extension of the model.) The anomalous dimension of φ^6 can be computed by considering the correlation function $\langle T_{\mu\nu}(x)\varphi^6(y)\rangle$ or, equivalently by considering $G_{66}(x,y) = \langle\varphi^6(x)\varphi^6(y)\rangle$, which satisfies the Callan-Symanzik equation

$$\left[\frac{\partial}{\partial\ln\mu} + \beta_g\frac{\partial}{\partial g} - 2\lambda_{(\varphi^6)}\right] G_{66} = 0\,. \tag{3.2}$$

At leading order the diagram



gives

$$G_{66} \sim 1 + \frac{20}{3!}g\int \frac{d^3k}{(2\pi)^3}\frac{d^3p}{(2\pi)^3}\,\frac{1}{k^2}\frac{1}{p^2}\frac{1}{(p+k)^2}\,, \tag{3.3}$$

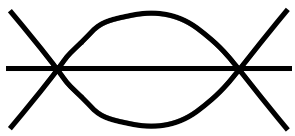
and thus

$$\lambda_{(\varphi^6)} = \frac{5}{3}\frac{g}{16\pi^2}\,. \tag{3.4}$$

The general relation (2.13) gives the β-function

$$\beta_g = \frac{5}{3}\frac{g^2}{16\pi^2}\,. \tag{3.5}$$

This result also follows from explicit computation of the diagram



which leads to the same integral as (3.3).

Inserting the results (3.5) and (3.4) for β and λ into (2.18) and (2.20) we find expressions for ϵ and η and hence for the CMB spectrum. We find at weak coupling $g \ll 1$

that $\epsilon \ll \eta$, in agreement with the general discussion in section 2.4. Of course in the explicit model the conformal dimension is derived in terms of the coupling g, a microscopic parameter.

We stress that these results should not be taken as a prediction of holography; they are the predictions derived from this specific dual theory which we have motivated only by its simplicity. The explicit discussion is meant to demonstrate how holographic considerations can lead to a concrete computational framework.

The theory (3.1) is not completely satisfying as an IR theory for several reasons. We have already mentioned that the existence of relevant perturbations lead to a fine tuning: this can be interpreted as a very particular renormalization group trajectory for which the action is valid. We will comment further on this issue in the following section. Another potential problem with this IR CFT is that it is weakly coupled: we of course chose this to be true to facilitate explicit computations, but it is presumably not expected to be the case, if the CFT is to be dual to a weakly coupled gravitational system.

4. Discussion

One of the motivations for considering holographic cosmology is that it offers an alternative viewpoint on several cosmological fine-tuning problems and may ultimately lead to their resolution. As a conclusion to the paper we discuss in turn the fine-tuning of the inflationary potential, the cosmological constant problem, and the initial value problem.

Fine-tuning of the inflationary potential. In the usual implementation of inflation, phenomenological constraints force the inflationary potential to be extremely flat. For example, in a model with $V(\varphi) = -\frac{1}{4}g\varphi^4$, one must have $g \sim 10^{-13}$ [25]. Such potentials are regarded as unnatural in quantum field theory: dimensionless coefficients are expected to be roughly of $\mathcal{O}(1)$. If one tries to choose them much smaller, they are expected to take their natural values after renormalization.

The holographic perspective improves this situation. Since we interpret the inflationary epoch as the final approach to the IR fixed point after a long flow from a complicated UV theory, there are naturally few degrees of freedom. Any fine tuning of the bulk theory has an interpretation in terms of the properties of this IR flow. A viable IR fixed point must possess suitable marginally irrelevant perturbations, and the RG flows must come in to the fixed point along these directions. The absence of relevant perturbations at the fixed point may be a desirable property from this point of view, although it is not clear if this is realistic. Modifications to the bulk inflaton potential do not correspond to quantum effects in the dual theory, but rather to changes in the properties of the fixed point. A nice property is that, in a given theory, universality of RG flows guarantees that the details of the UV region are not too important for inflation, apart from the fact that in a given model, particular trajectories may be required. This robustness of the IR theory thus appears to constitute a resolution of this fine-tuning problem.

The cosmological constant problem. In the holographic model the cosmological evolution is an inverse RG flow from small c_{IR} (large cosmological constant) in the past to

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large c_{UV} (small cosmological constant) in the future. The ratio

$$\frac{c_{UV}}{c_{IR}} = \frac{\Lambda_{IR}}{\Lambda_{UV}} \sim 10^{120}\,,$$

is enormous and the holographic perspective does not require this to be so (there are perfectly respectable RG flows with $c_{UV}/c_{IR} \sim$ a few). Holographic cosmology therefore does not resolve the cosmological constant problem.

On the other hand, a possible motivation for large c_{UV}/c_{IR} may be the following. In the AdS/CFT correspondence classical space-times correspond to the large N limit, and thus to a large value of the central charge. In the holographic scenario, the value of the cosmological constant is rather directly related to the complexity of the Universe, or more precisely, the complexity of the dual field theory. Large N in the AdS/CFT example may be a simple prototype for complexity in the Universe. There is thus a clear anthropic account of the cosmological constant here: only those Universes with sufficient complexity are realistic, and this corresponds to a large dual central charge, and a small cosmological constant.

A technical aspect of the usual cosmological constant problem is that small values are unstable to quantum corrections. It is important to contemplate the role of quantum corrections in the holographic scenario. Given a suitable starting point in the UV of the dual field theory, quantum effects simply generate the RG flow. The details of how large these corrections are do not matter in detail: the natural value for Λ is approached in the IR. One could also consider modifications to the UV theory, but as long as c_{UV} is large, they do not de-stabilize Λ. Thus, as long as the whole scenario exists, *the technical cosmological constant problem is resolved.* For other holographic discussions of the cosmological constant problem see [43, 44, 45].

The initial value problem. Inflationary models usually take the view that inflation was a finite epoch. Singularity theorems state that the spacetime preceding this epoch inevitably has a singularity somewhere [46], presumably to be resolved by string theory/M-theory. In principle, the pre-inflationary epoch provides the initial conditions for inflation; but in practice this data is inflated away, allowing inflation to make predictions that are independent of the initial conditions. The initial value problem is therefore not acute, but the situation is nevertheless conceptually unsatisfactory.

In holography the situation is quite different. In this paper, we have effectively assumed that the pre-inflationary period is eternal de Sitter, as the endpoint of the dual RG flow was assumed to be the IR fixed point. Technically, the singularity theorems do not apply to eternal de Sitter but it is far from clear that de Sitter constitutes a satisfactory boundary condition. One problem is that we must somehow determine amplitudes for all irrelevant operators in the theory. Another (presumably related) issue is that, after spatially inhomogeneous field configurations are taken into account, de Sitter may be unstable towards fragmentation [47, 48].

A variation of our scenario is to identify the inflationary epoch in the dual theory as a fortuitously close approach (with sufficient e-foldings) to a fixed point with relevant

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directions. If this were the case, then the pre-inflationary epoch would correspond to some further RG flow; and the properties of this flow would constitute everything there is to the initial value problem.

Of course, in either formulation, the initial values are observationally irrelevant. As we have shown, a single operator typically dominates, and we can make predictions without knowing its detailed properties. Thus the practical situation is completely parallel to standard inflation.

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